Finite-temperature Bell test for quasiparticle entanglement in the Fermi sea

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We demonstrate that the Bell test cannot be realized at finite temperatures in the vast majority of electronic setups proposed previously for quantum entanglement generation. This fundamental difficulty is shown to originate in a finite probability of quasiparticle emission from Fermi-sea detectors. In order to overcome the feedback problem, we suggest a detection strategy, which takes advantage of a resonant coupling to the quasiparticle drains. Unlike other proposals, the designed Bell test provides a possibility to determine the critical temperature for entanglement production in the solid state.

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It is well known that, unlike photons, quasiparticles in the Fermi sea injected from reservoirs, which are kept at thermal equilibrium, can be entangled by just a tunnel barrier.[1] This allows for particularly simple proposals for quantum quasiparticle entanglement, which do not involve interactions.[1–5] Theoretical results for the entanglement production in different electronic setups have been summarized in Refs. 6 and 7, while yet no experimental evidence of the quasiparticle entanglement in the Fermi sea has become available.

The quantum entanglement of two particles with respect to a spinlike degree of freedom can be accessed experimentally by measuring the spin correlator,

$$\mathcal{C}(\mathbf{a},\mathbf{b}) = \langle (\mathbf{a}\cdot\boldsymbol{\sigma})_1 \otimes (\mathbf{b}\cdot\boldsymbol{\sigma})_2 \rangle, \quad (1)$$

where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ is the vector of the Pauli matrices. The spin projection of the particles in the detectors 1 and 2 is measured with respect to the unit vectors **a** and **b** correspondingly. If the correlation between the particles is of a classical origin, the following Bell inequality holds,[8]

$$\mathcal{B} = |\mathcal{C}(\mathbf{a},\mathbf{b}) + \mathcal{C}(\mathbf{a}',\mathbf{b}) + \mathcal{C}(\mathbf{a},\mathbf{b}') - \mathcal{C}(\mathbf{a}',\mathbf{b}')| \le 2, \quad (2)$$

for arbitrary choice of the unit vectors **a**, **b**, **a**′, **b**′. The violation of the inequality [Eq. (2)] is, therefore, sufficient but not necessary condition for quantum entanglement.

In solid-state electronics, we deal with elementary excitations in the Fermi gas, which are referred to as quasiparticles. Even though the pairwise quasiparticle entanglement[9,10] is believed to be generated in many devices,[11–14] its experimental observation is obscured by the nature of electronic detectors. Those, unlike the photodetectors in optical setups, contain a number of quasiparticles in the ground state, which fill up available quantum levels below the Fermi energy. If a part of the device is at finite temperature, the electron and hole excitations are spontaneously created near the Fermi surface resulting in a finite probability for a Fermi-sea detector to emit. Such processes are harmful for any sensible Bell test.

The problem of quasiparticle entanglement detection has been put forward in Refs. 15 and 16, where the possibility to construct a Bell-type inequality with current cross correlators is discussed. It has been suggested to take advantage of the generalized spin correlator,

$$\mathcal{C}^M(\mathbf{a},\mathbf{b}) = \frac{\langle (N_{1\uparrow}-N_{1\downarrow})(N_{2\uparrow}-N_{2\downarrow})\rangle}{\langle (N_{1\uparrow}+N_{1\downarrow})(N_{2\uparrow}+N_{2\downarrow})\rangle}, \quad (3)$$

where $N_{n\sigma}$ is a number of particles with a spin projection σ registered by the detector n. (In solid state, the role of spin can be played by other quantum degrees of freedom such as orbital momentum or isospin.) Similarly to Eq. (1), the spin projection in Eq. (3) is measured with respect to the direction **a** in the first detector and **b** in the second one. Both definitions [Eqs. (1) and (3)] are equivalent in the original Bell setup, if no more than two particles are received within the detection time and the detectors do not emit particles. In electronic circuits, the number of quasiparticles $N_{n\sigma}$ is given by the time integral of a current $I_{n\sigma}$ flowing to the corresponding Fermi-sea reservoir,

$$N_{n\sigma} \propto \int_0^{t_{\rm det}} dt I_{n\sigma}(t), \quad (4)$$

which is not restricted. For large detection times $t_{\rm det}$, one typically observes $|N_{n\uparrow}-N_{n\downarrow}| \ll |N_{n\uparrow}+N_{n\downarrow}|$, hence the Bell inequality [Eq. (2)] cannot be violated and the corresponding measurement is useless for entanglement detection. The difficulty has been discussed in Ref. 16 for zero temperature.

An essential problem occurs in the opposite limit $t_{\rm det}\to 0$ because $N_{n\sigma}$ defined by Eq. (4) can take on negative values. This leads to fluctuations with $|N_{n\uparrow}-N_{n\downarrow}| > |N_{n\uparrow}+N_{n\downarrow}|$, which are explicitly forbidden in the Bell test. This situation is realized at finite temperatures. Then, the violation of Eq. (2) has no relation to the entanglement detection and the corresponding measurement is not of a Bell type.

Thus, the violation of the inequality [Eq. (2)] with the correlator $\mathcal{C}$ substituted by $\mathcal{C}^M$ does not provide a conclusive evidence for quantum entanglement generation at any finite temperature. This difficulty clearly applies to the detection of electron-hole entanglement[7,13] produced by tunneling events or by time-dependent gating. However, even in more sophis-

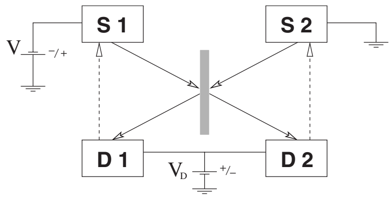

FIG. 1. A generic beam splitter for entanglement production in the solid state. The voltage bias applied between the sources S1 and S2 generates an entangled outgoing state at the detectors D1 and D2 provided the temperature in the sources T is smaller than a critical temperature T_c.

ticated setups where zero-temperature detectors and finite-temperature sources are represented by different metallic leads (in close resemblance to the original Bell proposal), the Bell test based on Eq. (3) is flawed. Examples include three-terminal fork geometries[4] and four-terminal beam-splitter geometries with grounded detectors. We focus on the latter (see Fig. 1) due to a number of previously proposed realizations,[1,3–5,11,17,18] which are mostly based on the directed transport along quantum-Hall edge channels. Minor modifications, such as lowering chemical potential in one of the detectors or increasing detection time, can suppress the probability of detector emission but lead, instead, to useless measurement with $|N_{n\uparrow}-N_{n\downarrow}| \ll |N_{n\uparrow}+N_{n\downarrow}|$. The generic situation is illustrated in Fig. 2 for the case of electronic beam splitter.

For quantum particles, the spin correlator from Eq. (3) is expressed through the expectation value,

$$\langle N_{1\sigma} N_{2\sigma'} \rangle \propto K_{\sigma\sigma'},$$

$$K_{\sigma\sigma'} = t_{\mathrm{det}}^2 \left[\langle I_{1\sigma} \rangle \langle I_{2\sigma'} \rangle + \int \frac{d\omega}{2\pi} \mathcal{P}_{\sigma\sigma'}(\omega) \mathcal{F}(\omega t_{\mathrm{det}}/2) \right], \quad (5)$$

where N and I are regarded as operators. We introduce the function $\mathcal{F}(x)=(\sin x)^2/x^2$ and the frequency-dependent cross correlator,

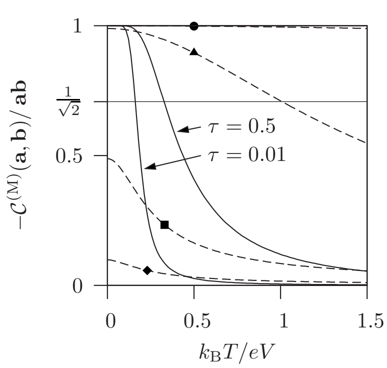

FIG. 2. The spin cross correlator $\mathcal{C}$ obtained from the density matrix of the final scattering state [solid lines; cf. Eq. (20)], and its generalization $\mathcal{C}^M$, evaluated numerically from Eqs. (9)–(15) for different values of the detection time $eVt_{\mathrm{det}}/h=0.01$ (●), 0.1 (▲), 1 (■), 5 (♦) [dashed lines; see Eqs. (21) and (22)].

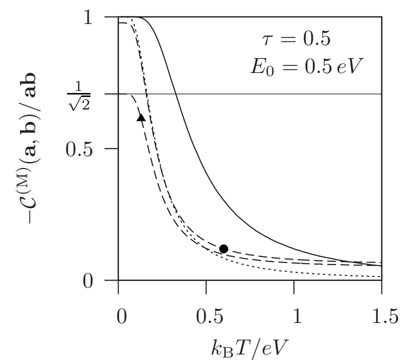

FIG. 3. The case of resonant detector coupling. The short-dashed line shows $\mathcal{C}^M$ from Eq. (26), while the long-dashed lines are numerical results for the Breit–Wigner resonances [Eq. (24)] with finite width $\Gamma=0.01eV$, detector voltage $V_D=-V$, and different detection times $\Gamma t_{\mathrm{det}}/h=0.01$ (●), 0.1 (▲). The measurement is useless for $t_{\mathrm{det}} \gtrsim 0.1h/\Gamma$. The solid line shows the correlator $\mathcal{C}$ from Eq. (20).

$$\mathcal{P}_{\sigma\sigma'}(\omega) = \int dt e^{i\omega t} \langle \delta I_{1\sigma}(t) I_{2\sigma'}(0) \rangle, \quad (6)$$

with $\delta I_{n\sigma}(t)=I_{n\sigma}(t)-\langle I_{n\sigma}\rangle$. In Figs. 2 and 3, the correlator $\mathcal{C}^M$ defined by experimentally measurable quantities [Eqs. (5) and (6)] is compared with the exact result of the density matrix analysis of the final state.[7]

We restrict ourselves to an important class of systems which do not involve spin-dependent scattering because the chances to generate quantum entanglement with respect to the spin degree of freedom are obviously maximized in such setups. The values of $\mathcal{P}_{\sigma\sigma'}$ in Eq. (6) are related to the cross correlator $\mathcal{P}$ of the corresponding spin-independent problem as

$$\mathcal{P}_{\uparrow\uparrow} = \mathcal{P}_{\downarrow\downarrow} = \frac{1}{2}(1+\mathbf{ab})\mathcal{P}, \quad (7)$$

$$\mathcal{P}_{\uparrow\downarrow} = \mathcal{P}_{\downarrow\uparrow} = \frac{1}{2}(1-\mathbf{ab})\mathcal{P}. \quad (8)$$

This symmetry holds even for interacting electronic systems provided the absence of spin dephasing. It follows from Eqs. (7) and (8) that a neglection of the mean currents[5,7] in Eq. (5) is equivalent to $\mathcal{C}^M(\mathbf{a},\mathbf{b})=\mathbf{ab}$, hence the inequality [Eq. (2)] is violated with $\mathcal{B}_{\max}=2\sqrt{2}$ irrespective of voltages, temperature, or other setup characteristics. Clearly, such violation has nothing to do with pairwise quantum entanglement. We will see that the problem persists even if the exact expression for $K_{\sigma\sigma'}$ is used.

In the absence of spin-dependent scattering, the mean currents measured by the detectors do not depend on the directions $\mathbf{a}$ and $\mathbf{b}$, $\langle I_{n\sigma}\rangle=\langle I_n\rangle$. From Eqs. (5)–(8), we obtain

$$\mathcal{C}^M(\mathbf{a},\mathbf{b}) = \frac{\gamma}{2+\gamma}\mathbf{ab}, \quad (9)$$

with a parameter γ given by the ratio

$$\gamma = \frac{\int d\omega \mathcal{P}(\omega)\mathcal{F}(\omega t_{\rm det}/2)}{2\pi\langle I_1\rangle\langle I_2\rangle}, \qquad (10)$$

where both the cross correlator $\mathcal{P}(\omega)$ and the product of the mean currents have to be calculated for the corresponding spin-independent problem.

An example of such a calculation can be performed within the Landauer–Büttiker scattering approach,[19] which is valid as far as inelastic processes in between the reservoirs can be disregarded. Within the scattering approach, the mean current to the reservoir α is given by

$$\langle I_\alpha\rangle = \frac{e}{h}\int dE \sum_\beta (\delta_{\alpha\beta} - |S_{\alpha\beta}(E)|^2) f_\beta(E), \qquad (11)$$

where $f_\alpha(E)=(1+\exp[(E-eV_\alpha)/k_BT_\alpha])^{-1}$ is the Fermi distribution function, which depends on the temperature of the corresponding reservoir T_α and the voltage bias V_α applied. Frequency-dependent correlator (6) of the currents flowing to the reservoirs α and α' reads[19]

$$\mathcal{P}_{\alpha\alpha'}(\omega) = \frac{e^2}{2h}\int dE \sum_{\beta\beta'} M_{\alpha\alpha',\beta\beta'}(E,\hbar\omega) F_{\beta\beta'}(E,\hbar\omega),$$

$$F_{\beta\beta'}(E,\Omega) = f_\beta(E)\tilde{f}_{\beta'}(E+\Omega) + \tilde{f}_\beta(E) f_{\beta'}(E+\Omega),$$

$$M_{\alpha\alpha';\beta\beta'}(E,\Omega) = [\delta_{\alpha\beta}\delta_{\alpha\beta'} - S^*_{\alpha\beta}(E)S_{\alpha\beta'}(E+\Omega)] \times [\delta_{\alpha'\beta}\delta_{\alpha'\beta'} - S^*_{\alpha'\beta'}(E+\Omega)S_{\alpha'\beta}(E)], \qquad (12)$$

$$\tilde{f}(E) \equiv 1 - f(E). \qquad (13)$$

Let us consider a generic beam splitter with no spin-dependent scattering depicted schematically in Fig. 1. Such a setup is characterized by an energy-independent S matrix,

$$S = \begin{pmatrix} 0 & s' \\ s & 0 \end{pmatrix}, \qquad (14)$$

where 2×2 unitary matrices s and s' describe the transport from sources to detectors and from detectors to sources, correspondingly. We parametrize

$$s = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi'} \end{pmatrix}\begin{pmatrix} \sqrt{1-\tau} & i\sqrt{\tau} \\ i\sqrt{\tau} & \sqrt{1-\tau} \end{pmatrix}\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta'} \end{pmatrix}, \qquad (15)$$

where $\tau\in[0,1]$ is the beam-splitter transparency and the spin index is omitted. Following the majority of proposals, both detectors and the second source are grounded, i.e., $V_{\rm D} \equiv V_{\rm D1}=V_{\rm D2}=0$, $V_{\rm S2}=0$, while $V_{\rm S1}=V$ is the voltage applied between the sources.

At zero temperature, the beam splitter acts[7] as a source of spin-entangled Bell pairs,

$$|\Psi_B\rangle = \frac{1}{\sqrt{2}}|\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\rangle, \qquad (16)$$

where the index $n=1,2$ refers to the detector number. Such an entanglement generation is due to the Pauli principle, which guarantees that a filled state with $E\in(0,eV)$ in the first source contains exactly two quasiparticles with the opposite spins.

The Bell pairs can be accessed at zero temperature by performing a time-coincidence detection. For finite temperature T in the sources, the density matrix projection, which corresponds to a single particle in each detector, is derived in Appendix A,[7]

$$\rho_{11}^{\rm out} = \frac{1}{4}(1-\xi)\mathbb{1}_4 + \xi|\Psi_B\rangle\langle\Psi_B|, \qquad (17)$$

where $\mathbb{1}_4$ is the unit matrix in the two-particle Hilbert space and ξ is an energy-independent weight factor,

$$\xi = \frac{\tau(1-\tau)(f_{\rm S1}-f_{\rm S2})^2}{\tau(1-\tau)(f_{\rm S1}-f_{\rm S2})^2 + 2f_{\rm S1}\tilde{f}_{\rm S1}f_{\rm S2}\tilde{f}_{\rm S2}}, \qquad (18)$$

where $f_{{\rm S}n}$ is the Fermi distribution function in nth source. Result (17) describes the mixed Werner state,[20] which is entangled as far as $\xi>1/3$ according to the Wootters formula.[21] In the present case, this condition is equivalent to $T<T_c$ with the critical temperature T_c determined by the equation[7]

$$\tau(1-\tau)\sinh^2(eV/2k_BT_c) = 1/4. \qquad (19)$$

From Eqs. (1) and (17), one obtains the exact spin correlator,

$$\mathcal{C}(\mathbf{a},\mathbf{b}) = -\xi\mathbf{ab}, \qquad (20)$$

which is plotted in Fig. 2 with the solid line for different values of the transparency parameter. The corresponding Bell inequality [Eq. (2)] can be violated for $\xi>1/\sqrt{2}$, which is, indeed, a sufficient condition for the entanglement. Whether or not such a Bell test can be performed by measuring current cross correlator [Eq. (3)] is, however, an open question.

In order to answer this question, we substitute expression (14) for the S matrix to Eqs. (11) and (12), where the summation runs over the index β={S1,S2,D1,D2}. The correlator $\mathcal{C}^M$ is, then, obtained from Eqs. (9) and (10) with $I_1 \equiv I_{\rm D1}$, $I_2\equiv I_{\rm D2}$, and $\mathcal{P}\equiv\mathcal{P}_{\rm D1,D2}$.

For $t_{\rm det}\gg\min\{h/eV, h/k_BT\}$, we obtain

$$\gamma = -\frac{h}{eVt_{\rm det}}\left[\coth\left(\frac{eV}{2k_BT}\right) - \frac{2k_BT}{eV}\right] \ll 1, \qquad (21)$$

i.e., the corresponding measurement is useless for an entanglement detection. Indeed, such a long-time measurement is not projective; therefore, it does not single out the state with one quasiparticle in each detector.[22]

In the opposite limit, we, however, find

$$\gamma = -1, \quad t_{\rm det} \ll \min\{h/eV, h/k_BT\}, \qquad (22)$$

hence, the inequality [Eq. (2)] is violated for any temperature of the source. Thus, according to the density matrix analysis [Eqs. (17) and (20)], the corresponding measurement is not of a Bell type. Both results (21) and (22) formally hold for any temperature of the detectors.

The transition from non-Bell-type measurement to the useless measurement with the increase of $t_{\rm det}$ is illustrated in Fig. 2. The Bell parameter defined with the correlator $\mathcal{C}^M$ does not depend on the beam-splitter transparency τ and can easily exceed 2 even in the absence of any entanglement.

The result of Eq. (22) is equivalent to

$$\langle I_{\rm D1}(t)I_{\rm D2}(t)\rangle = 0. \tag{23}$$

At $T=0$, the currents $I_{{\rm D}n}(t)$ are sign definite; hence, Eq. (23) is exact for every single time-coincidence measurement in agreement with the prediction of the density matrix approach. For rising temperatures $T>0$, the correlation [Eq. (23)] holds only on average and is not sensitive to vanishing quantum entanglement in the final state of the beam splitter [Eq. (17)]. Consequently, the inequality [Eq. (2)] with $\mathcal{C}$ substituted by $\mathcal{C}^M$ can be violated for arbitrarily high temperatures. The absence of critical temperature indicates once again[23] that such a violation has nothing to do with the entanglement detection. Instead, the decay of $\mathcal{C}^M(\mathbf{a},\mathbf{b})$ with the temperature in Fig. 2 (dashed lines) is determined by the detection time $t_{\rm det}$. Thus, the measurement of $\mathcal{C}^M(\mathbf{a},\mathbf{b})$ cannot be used for the entanglement test in the beam-splitter setup and the value of T_c cannot be inferred from such a measurement as the matter of principle.

We propose a way to rescue the Bell measurement by coupling detectors via the energy filters, which are described by energy-dependent scattering amplitudes: r_n, r_n', t_n, t_n', where $n=1,2$ is the number of the detector. The use of energy filters in the context of Bell measurement at zero temperature has been discussed in Ref. 24. Let us illustrate our results for the case of identical filters with the Breit–Wigner form of the transmission amplitude,

$$t_n(E) = e^{i\delta_n}(\Gamma/2)(E-E_0-i\Gamma/2)^{-1}. \tag{24}$$

The S matrix of the full setup including the filters is given by

$$S(E) = \begin{pmatrix} s' r(E) s & s' t'(E) \\ t(E) s & r'(E) \end{pmatrix}, \tag{25}$$

where $t=\mathrm{diag}(t_1,t_2)$, $r=\mathrm{diag}(r_1,r_2)$, etc. The condition for time-coincidence detection now reads $t_{\rm det} \ll h/\Gamma$. The currents $I_{{\rm D}n}(t)$ can be made sign definite by applying an additional voltage bias $V_{\rm D}$, as shown in Fig. 1. The current fluctuations due to temperature are not harmful for the Bell test as far as $|eV_{\rm D}| \gg k_B T$, which is the only restriction on the value of $V_{\rm D}$. In this case, there is no requirement for an additional cooling of the detectors, so that a whole setup can be kept in temperature equilibrium. Moreover, for $\Gamma \ll eV$, the dependence on $t_{\rm det}$ vanishes, meaning that $\mathcal{C}^M(\mathbf{a},\mathbf{b})$ can be obtained experimentally from zero-frequency noise measurements. The feasibility of such a Bell test is illustrated in Fig. 3 for realistic values of the parameters.

For $\Gamma\to 0$, we obtain, from Eqs. (9), (10), (12), (24), and (25), that

$$\mathcal{C}^M = -\left.\frac{\tau(1-\tau)(f_{\rm S1}-f_{\rm S2})^2\mathbf{ab}}{\tau(1-\tau)(f_{\rm S1}-f_{\rm S2})^2 + 2f_{\rm S1}f_{\rm S2}}\right|_{E=E_0}. \tag{26}$$

The result is plotted with the short-dashed line in Fig. 3. It is evident from the comparison with Eqs. (18) and (20) that the proposed measurement is always of the Bell type. The correlator [Eq. (26)] tends to the exact one [Eq. (20)] for $E_0 \gg eV$. The setup efficiency is, however, exponentially low in this limit. The numerical results in the case of finite resonance width $\Gamma=0.01eV$ are plotted in Fig. 3 with the dashed lines. The test provides the lower estimate for the critical temperature.

In conclusion, we point out the fundamental restrictions for the Bell test in electronic setups due to the quasiparticle emission from Fermi-sea detectors. We propose a way to rescue the Bell measurement by a resonant coupling to the detectors. We show that the lower estimate of the critical temperature for entanglement production can be experimentally obtained in the proposed setup.

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APPENDIX A: DENSITY MATRIX PROJECTION

Following Ref. 7, we review the derivation of the density matrix projection [Eq. (17)] for final scattering state in the case of the setup depicted in Fig. 1. The density matrix of the incoming state is given by

$$\rho^{\rm in} = \prod_{n,E,\sigma} (\tilde{f}_{{\rm S}n}(E)|0\rangle\langle 0| + f_{{\rm S}n}(E)a^\dagger_{n\sigma E}|0\rangle\langle 0|a_{n\sigma E}), \tag{A1}$$

where $f_{{\rm S}n}$ is the Fermi distribution function in the source Sn, $\tilde{f}_{{\rm S}n} \equiv 1-f_{{\rm S}n}$, and $a_{n\sigma E}$ is the fermion annihilation operator for an incoming scattering state at the channel n and energy E. The annihilation operators for the outgoing scattering states, $b_{n\sigma E}$, are obtained from the relation,

$$b_{n\sigma E} = \sum_m s_{nm}(E)a_{m\sigma E}, \tag{A2}$$

where s_{nm} are the components of a unitary scattering matrix.

Thus, the density matrix of the final state is

$$\rho^{\rm out} = \prod_{n,E,\sigma} \{\tilde{f}_{{\rm S}n}(E)|0\rangle\langle 0| + f_{{\rm S}n}(E)c^\dagger_{n\sigma E}|0\rangle\langle 0|c_{n\sigma E}\}, \tag{A3}$$

where

$$c^\dagger_{n\sigma E} = \sum_m b^\dagger_{m\sigma E}s_{mn}(E). \tag{A4}$$

In order to quantify the two-particle entanglement for the partition $\mathcal{H}_{\rm D1}\otimes\mathcal{H}_{\rm D2}$ of the Hilbert space with respect to the detectors, the state $\rho^{\rm out}$ has to be projected onto the sectors $\mathcal{N}_{E_1N_1,E_2N_2}$ of the Fock space with the energies E_1,E_2 and particle numbers N_1,N_2 in the corresponding detectors D1, D2. The density matrix $\rho^{\rm out}_{N_1,N_2}$ of the projection factorizes into a product state in all sectors except for the sector $\mathcal{N}_{E_1,E_1}$, with $E_1=E_2=E$ and $N_1=N_2=1$. Projection onto this sector is found from Eq. (A3) as

$$w_{11}\rho_{11}=f_{S1}\tilde{f}_{S1}f_{S2}\tilde{f}_{S2}\mathbb{1}_4+2\tau(1-\tau)(f_{S1}-f_{S2})^2|\Psi_B\rangle\langle\Psi_B|, \tag{A5}$$

where $\mathbb{1}_4$ is the unit matrix in the two-particle Hilbert space, $|\Psi_B\rangle$ is the Bell state [Eq. (16)], and the weight factor w_{11} is determined from the condition Tr $\rho_{11}=1$ as

$$w_{11}=4f_{S1}\tilde{f}_{S1}f_{S2}\tilde{f}_{S2}+2\tau(1-\tau)(f_{S1}-f_{S2})^2. \tag{A6}$$

From Eqs. (A5) and (A6), we obtain Eqs. (17) and (18). By substituting $f_{Sn}=(1+\exp[(E-eV_n)/k_BT])$, with $V_1=V$, $V_2=0$ in Eq. (18), we can further simplify the parameter ξ as

$$\xi(T)=1-\left[1+2\tau(1-\tau)\sinh^2\frac{eV}{2k_BT}\right]^{-1}. \tag{A7}$$

The critical temperature T_c is determined from the equation $\xi(T_c)=1/3$, which is equivalent to Eq. (19).

APPENDIX B: EVALUATION OF THE CORRELATOR $\mathcal{C}^M$

1. Plain beam splitter

We evaluate the generalized spin correlator $\mathcal{C}^M(\mathbf{a},\mathbf{b})$ given by Eqs. (9) and (10) in the framework of the scattering approach. By substituting the scattering matrix [Eqs. (14) and (15)] into Eq. (11), we calculate the mean currents, which are measured in the detectors D1, D2, as

$$\langle I_{D1}\rangle=-\frac{e}{h}(1-\tau)eV, \quad \langle I_{D2}\rangle=-\frac{e}{h}\tau eV. \tag{B1}$$

The cross correlator [Eq. (12)] is found as

$$\mathcal{P}_{D1,D2}(\omega)=\frac{e^2}{2h}\tau(1-\tau)\left[2\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)-\sum_{\zeta=\pm1}(eV+\zeta\hbar\omega)\coth\left(\frac{eV+\zeta\hbar\omega}{2k_BT}\right)\right]. \tag{B2}$$

The parameter γ given by Eq. (10) can be calculated analytically in two opposite limits.

(i) For large detection times, $t_{det}\gg\min\{h/eV,h/k_BT\}$, one can replace $t_{det}\mathcal{F}(\omega t_{det}/2)$ with $2\pi\delta(\omega)$, hence

$$\gamma=\frac{\mathcal{P}_{D1,D2}(0)}{t_{det}\langle I_{D1}\rangle\langle I_{D2}\rangle}. \tag{B3}$$

This leads to result (21).

(ii) For short detection times, $t_{det}\ll\min\{h/eV,h/k_BT\}$, one can approximate $\mathcal{F}(\omega t_{det}/2)\approx1$ in the relevant frequency range $|\hbar\omega|\lesssim eV$. In this limit, the integral in Eq. (10) does not depend on temperature,

$$\int d\omega\mathcal{P}_{D1,D2}(\omega)=-2\pi\left(\frac{e^2V}{h}\right)^2\tau(1-\tau), \tag{B4}$$

which leads to simple result (22).

2. Beam splitter with energy filters

We repeat the calculation in a more general case of an energy-dependent scattering matrix [Eq. (25)]. From Eq. (11), we obtain the mean currents,

$$\langle I_{D1}\rangle=\frac{e}{h}\int dE|t_1(E)|^2[f_D(E)-(1-\tau)f_{S1}(E)-\tau f_{S2}(E)], \tag{B5}$$

$$\langle I_{D2}\rangle=\frac{e}{h}\int dE|t_2(E)|^2[f_D(E)-\tau f_{S1}(E)-(1-\tau)f_{S2}(E)], \tag{B6}$$

where $f_D(E)=(1+\exp[(E-eV_D)/k_BT])$ is the Fermi distribution function in the detectors. From Eq. (12), we calculate the cross correlator,

$$\mathcal{P}_{D1,D2}(\omega)=\frac{e^2}{2h}\tau(1-\tau)\int dEt_1^*(E)t_1(E+\hbar\omega)t_2^*(E+\hbar\omega)t_2(E)\times[F_{S1,S1}(E,\hbar\omega)+F_{S2,S2}(E,\hbar\omega)-F_{S1,S2}(E,\hbar\omega)-F_{S2,S1}(E,\hbar\omega)], \tag{B7}$$

where the function $F_{\alpha\beta}$ is defined in Eq. (12). These expressions allow for the numerical evaluation of $\mathcal{C}^M(\mathbf{a},\mathbf{b})$ for arbitrary energy-dependent scattering matrix [Eq. (25)].

In the case of sharp resonances, such as those of the Breit–Wigner form [Eq. (24)] with $\Gamma\to0$, we have

$$\lim_{\Gamma\to0}\Gamma^{-1}|t_n(E)|^2=\frac{\pi}{2}\delta(E-E_0), \tag{B8}$$

and obtain, from Eqs. (B5)–(B8),

$$\gamma=\frac{-\tau(1-\tau)(f_{S1}-f_{S2})^2}{[f_D-(1-\tau)f_{S1}-\tau f_{S2}][f_D-\tau f_{S1}-(1-\tau)f_{S2}]}, \tag{B9}$$

where the Fermi functions are evaluated at the position of the resonance $E=E_0>0$. For $|eV_D|\gg k_BT$, the value $f_D(E_0)$ is exponentially small, hence Eq. (26) is justified. In general, the setup is functional provided the detector voltage V_D is sufficiently large to ensure that f_D is much smaller than both f_{S1} and f_{S2} within the energy window of the filters.

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